Exercise 336

For the following problems, consider radioactive dating. A human skeleton is found in an archeological dig. Carbon dating is implemented to determine how old the skeleton is by using the equation $y = e^{rt}$, where y is the ratio of radiocarbon still present in the material, t is the number of years passed, and $r = -0.0001210$ is the decay rate of radiocarbon.

Find the inverse of the carbon-dating equation. What does it mean? If there is 25% radiocarbon, how old is the skeleton?

Solution

Start by switching y with t in the given equation.

$$t = e^{ry}$$

Solve for y by taking the natural logarithm of both sides.

$$\ln y = \ln e^{rt}$$

Use the property of logarithms that allows the exponent of the argument to be brought down in front.

$$\ln y = (rt) \ln e$$

Use the fact that $\ln e = 1$.

$$\ln y = rt$$

Divide both sides by r.

$$t = \frac{1}{r} \ln y$$

This is the inverse function: It tells us how many years have passed for a given ratio of radiocarbon remaining. For example, if there's 25% radiocarbon remaining, then $y = 0.25$ and

$$t = \frac{1}{-0.0001210} \ln 0.25 \approx 11\,457,$$

which means the skeleton is 11,457 years old.